Quadratic Equations and Complex Numbers

Chapter Review

Solving by Taking Square Roots

- Always two solutions in quadratic equations
- Can not take the square-root of a negative number - in the Real Number system

Example,

$\sqrt{9}$ → $+3$, -3

$\sqrt{-9}$ ← Not a Real Number - the Answer is a Complex number

Steps

1. Isolate the "x^2" term
2. Take the square-root of both sides

Example

Step 1

$$2x^2 - 4 = 10$$
$$\underline{\quad +4 \quad +4}$$
$$\frac{2x^2}{2} = \frac{14}{2}$$
$$x^2 = 7$$

Step 2

$$\sqrt{x^2} = \sqrt{7}$$
$$x = \pm\sqrt{7}$$

← two solutions + and −

Graphing Quadratic Equations

Steps

1. Find the vertex
2. Graph – the shape is always a parabola ("U"-shape)

Example $y = 2x^2 + 4x - 8$

Finding vertex – write equation is standard form (highest → lowest power, $ax^2 + bx + c = 0$)

$y = 2x^2 + 4x - 8$

$a = 2 \quad b = 4 \quad c = -8$

vertex is located at $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

Find $\frac{-b}{2a} = \frac{-(4)}{2(2)} = \frac{-4}{4} = -1$

Find $f\left(\frac{-b}{2a}\right) \rightarrow$ plug in $\frac{-b}{2a}$ into equation

$y = 2x^2 + 4x - 8$

$= 2(-1)^2 + 4(-1) - 8 = 2(1) + -4 - 8 = -10$

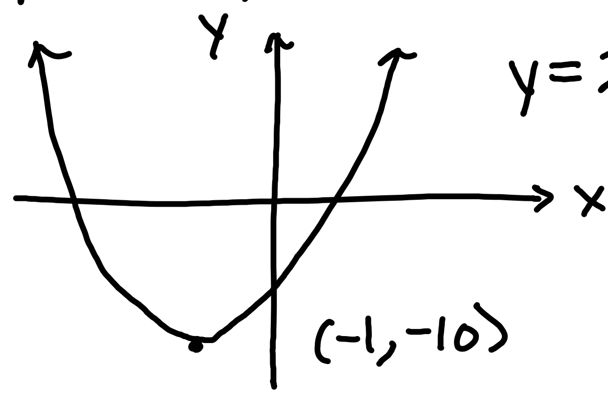
vertex $(-1, -10)$, $y = 2x^2 + 4x - 8$

x^2 term is positive – graph upward parabola

x^2 term negative – downward

Graph upward parabola from $(-1,-10)$

$$y = 2x^2 + 4x - 8$$



Quadratic Formula

- Very important! Need to master it to solve quadratic equations

When you have $ax^2 + bx + c = 0$

the solutions are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example solve

$$x^2 - 9x = -8$$

First write in standard form

$$1x^2 - 9x + 8 = 0$$

$a = 1 \quad b = -9 \quad c = 8$

plug in values into formula

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(8)}}{2(1)}$$

$$x = \frac{9 \pm \sqrt{81 - 32}}{2}$$

$$x = \frac{9 \pm \sqrt{49}}{2}$$

$$x = \frac{9+7}{2} \qquad x = \frac{9-7}{2}$$

$$x = \frac{16}{2} \qquad x = \frac{2}{2}$$

solutions $\rightarrow$ $x = 8$ $\quad$ $x = 1$

Solve Quadratic Equations by Factoring

- Need to know how to factor polynomials
- Only works when you can factor – otherwise use quadratic formula
- Based on zero-product property

Example $x^2 - 9x + 8 = 0$ ← Factor first

$$(x-1)(x-8) = 0$$

Zero product property: One of these terms must be zero, because thats the only way you can the equation (left side) can be equal to zero (right side)

Solve by setting both factors equal to zero

$$x - 1 = 0 \qquad x - 8 = 0$$

$$x = 1 \qquad x = 8$$ ← solutions

The Discriminant- Type of Roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

← the $b^2 - 4ac$ part is the Discriminant

When:

$b^2 - 4ac = 0$ → 1 double root

$b^2 - 4ac =$ positive → 2 real roots

$b^2 - 4ac =$ negative → No real roots

Note: "root" – means solutions

Completing the Square

- A method to rewrite a quadratic equation in such a way as to solve by taking the square-root of both sides

Example

1. write in standard form ($ax^2+bx+c=0$)

$$x^2-9x+8=0$$

move c to right side

$$x^2-9x=-8$$

add $\left(\frac{b}{2}\right)^2$ to both sides

$$x^2-9x+\left(-\frac{9}{2}\right)^2=-8+\left(-\frac{9}{2}\right)^2$$

$$x^2-9x+\frac{81}{4}=-8+\frac{81}{4}$$

Factor

$$\left(x-\frac{9}{2}\right)^2=\frac{49}{4}$$

Solve by taking $\sqrt{\ }$ both sides

$$\sqrt{\left(x-\frac{9}{2}\right)^2}=\sqrt{\frac{49}{4}}$$

$$x-\frac{9}{2}=\pm\frac{7}{2}$$

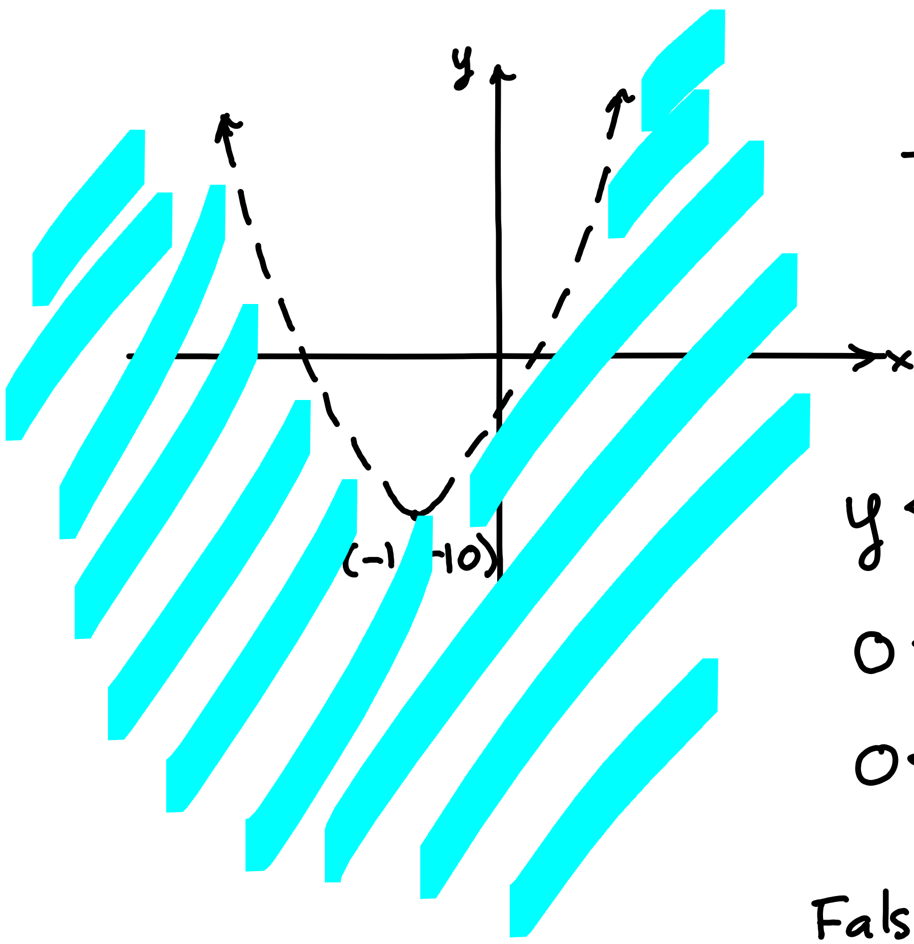
$$x-\frac{9}{2}=\frac{7}{2} \qquad\qquad x-\frac{9}{2}=-\frac{7}{2}$$

$$x=\frac{7}{2}+\frac{9}{2}=\frac{16}{2}=8 \qquad\qquad x=-\frac{7}{2}+\frac{9}{2}=\frac{2}{2}=1$$

Solutions

Graphing Quadratic Inequalities

Graph using same steps as linear inequalities



$y < 2x^2 + 4x - 8$

boarder is - - - -

Test a point, I like (0,0)

$y < 2x^2 + 4x - 8$

$0 < 2(0)^2 + 4(0) - 8$

$0 < -8$

False statement - shade under parabola

Complex Numbers

Complex Numbers – Number system
Real + imaginary

imaginary numbers have "i", $i = \sqrt{-1}$

$$i = \sqrt{-1} \quad \text{so} \quad \sqrt{-4} = \sqrt{4} \cdot \sqrt{-1} = \sqrt{4} \cdot i = \pm 2i$$

Complex Numbers – have two parts

$$\underbrace{a}_{\substack{\text{Real} \\ \text{Part}}} + \underbrace{bi}_{\substack{\text{imaginary} \\ \text{Part}}}$$

examples, $3 + 7i$, $-1 - 5i$, $12 + 1.9i$

Complex Number operations

add / subtract – like parts (same as like terms)

$$(3 + 7i) + (2 + 3i) = (3 + 2) + (7i + 3i)$$
$$= 5 + 10i$$

multiply use F.O.I.L. (same as binomial multiplication)

Use FOIL

$$(2+3i)(6+2i) = 12 + 4i + 18i + 6i^2$$

$$= 12 + 22i + 6i^2$$

$$i = \sqrt{-1}$$
$$i^2 = -1$$

$$= 12 + 22i + 6(-1)$$
$$= 12 + 22i + -6$$
$$(2+3i)(6+2i) = 6 + 22i$$

Complex Conjucate – how we remove a complex number from the denominator of a fraction (just like "rationalizing")

Real Numbers

$$\frac{2}{3+\sqrt{5}} \cdot \frac{(3-\sqrt{5})}{(3-\sqrt{5})}$$

conjucate

Complex Numbers

$$\frac{2}{3+6i} \cdot \frac{(3-6i)}{(3-6i)}$$

complex conjucate

Graphing Complex Numbers (very much like the Real Number (x, y) plane)

(Real, img)